HAT

Inverse Matrices

9/12/17

Warm Up:

4. Consider the matrices $A = \begin{bmatrix} 1 & -2 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 4 \\ 2 & 3 \end{bmatrix}$

a. Find the product. Show your work.

b. Find a matrix C such that $B + C = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

c. Find a matrix D such that $B \cdot D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ Identity Matrix

$$\begin{bmatrix} 4 & 4 \\ 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$4x + 4z = 1$

$-2 \cdot (2x + 3z = 0)$

$-4x - 6z = 0$

$-2z = 1$

$z = -\frac{1}{2}$

$4x - 2 = 1$

$4x = 3$

$x = \frac{3}{4}$

$4y + 4w = 0$

$(2y + 3w = 1)(-2)$

$-4y - 6w = -2$

$-2w = -2$

$w = 1$

$4y + 4 = 0$

$4y = -4$

$y = -1$

$$D = \begin{bmatrix} \frac{3}{4} & -1 \\ -\frac{1}{2} & 1 \end{bmatrix}$$

Example #1

a. Determine whether $X = \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}$ and $Y = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$ are inverses of each other.

Y·Z

$\begin{bmatrix} 1 & 0 \\ -6 & \end{bmatrix}$

NO

b. Determine whether $P = \begin{bmatrix} 3 & -1 \\ 4 & -2 \end{bmatrix}$ and $Q = \begin{bmatrix} 1 & -3 \\ 2 & 4 \end{bmatrix}$ are inverses of each other.

P·Q

$\begin{bmatrix} 1 & -5 \\ & \end{bmatrix}$

NO

determinant

- $\det D$
- $\det \begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \end{bmatrix}$
- $\begin{vmatrix} \cdot & \cdot \\ \cdot & \cdot \end{vmatrix}$

matrix D

$$\begin{bmatrix} a & -b \\ -c & d \end{bmatrix}$$

$ad - bc$

Inverse

$$D^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Example #2: Find the inverse.

a. $\begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix}$

det $6 - 0$

6

$\frac{1}{6}\begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}$

$\begin{bmatrix} \frac{1}{2} & \frac{1}{6} \\ 0 & \frac{1}{3} \end{bmatrix}$

b. $\begin{bmatrix} 5 & -10 \\ -1 & 2 \end{bmatrix}$

det $= 0$

No Inverse

$\frac{1}{0}\begin{bmatrix} & \\ & \end{bmatrix}$

Example #3: Solve the system using the matrix method.

$$\begin{cases} 4x - 5y = 28 \\ 9x + 7y = -10 \end{cases}$$

$2\times 2 \cdot 2\times 1$

2×1

$$\begin{bmatrix} 4 & -5 \\ 9 & 7 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 28 \\ -10 \end{bmatrix}$$

$$A^{-1} = \frac{1}{73}\begin{bmatrix} 7 & 5 \\ -9 & 4 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \frac{7}{73} & \frac{5}{73} \\ -\frac{9}{73} & \frac{4}{73} \end{bmatrix}$$

$$A \cdot X = S$$

$$A^{-1} \cdot A \cdot X = A^{-1} \cdot S$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot X =$$

$$X = \begin{bmatrix} \frac{7}{73} & \frac{5}{73} \\ -\frac{9}{73} & \frac{4}{73} \end{bmatrix} \cdot \begin{bmatrix} 28 \\ -10 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{196}{73} - \frac{50}{73} \\ -\frac{252}{73} - \frac{40}{73} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -4 \end{bmatrix}$$

Homework: pg. 202 #8, 21, 26, 31, 36